647510 **0301107C1010** **Seat No :________**

B.Sc. Semester - 1 (*CBCS*) Examination
Nov./Dec. -2018 (Old Course)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer the following questions. **[04]**

(1) GLB (0, 1) = __________ .

(2) True / False : Roll's theorem is a particular case of Langrange's theorem.

(3) Write formula of Cauchy's theorem.

(4) Write coefficient of x^3 in the expansion of $\cos x$ in terms of x.

Q.1 (B) Answer any ONE of the following questions. **[02]**

(1) Find $c \in (0, 1)$ by applying Lagrange's theorem to the function $f(x) = x^2$.

(2) Write Maclaurin's series expansion of (a) e^x and (b) $\sin x$.

Q.1 (C) Answer any ONE of the following questions. **[03]**

(1) For $x > 0$ show that $\frac{x}{1+x^2} < \tan^{-1}x < x$.

(2) Show that the function $f(x) = 3x^3 - 9x^2 + 9x + 7$ is strictly increasing on R.

Q.1 (D) Answer any ONE of the following questions. **[05]**

(1) State and prove Roll's mean value theorem.

(2) Let $f(x) = a^x$, $g(x) = \tan^{-1}(x)$; $x \in [0, b]$;$(0 < a < b)$. Show that there exists $c \in (0, b)$ such that $\frac{a^b - 1}{\log a} = a^c(1 + c^2)\tan^{-1}b$.

Q.2 (A) Answer the following questions. **[04]**

(1) $\lim_{x \to \frac{\pi}{2}} \frac{\log \sin x^n}{\cos^2 x}$.

(2) Form a differential equation from $y = a \sin bx$, where a and b are constants.

(3) Write general form of Linear differential equation.

(4) Solve : $\frac{dy}{dx} = \frac{x}{y}$.

Q.2 (B) Answer any ONE of the following questions. **[02]**

(1) Find I.F. of the differential equation $x \log x \frac{dy}{dx} + y = 2x$.

(2) Evaluate : $\lim_{x \to 0} \left(\frac{\cos x - 1 + \frac{x^2}{2}}{x^4} \right)$.

Q.2 (C) Answer any ONE of the following questions. **[03]**

(1) Solve : $e^x\, dy + (ye^x + 2x)dx = 0$.

(2) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3} = b$, then find values of a and b.

Q.2 (D) Answer any ONE of the following questions. **[05]**

(1) Solve : $(x^2 + y^2)\, dx + 2xy\, dy = 0$.

(2) Evaluate : $\lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$.

Q.3 (A) Answer the following questions. **[04]**

(1) True / False : Bernouli's differential equation is a particular case of linear differential equation.

(2) Write solution of the differential equation $y = px + \log p$.

(3) Write general form of Lagrange's differential equation.

(4) True / False : Clairaut's differential equation is a particular case of Lagrange's differential equation.

Q.3 (B) Answer any ONE of the following questions. **[02]**

(1) Solve : $x \frac{dy}{dx} + y = x^3 y^6$.

(2) Solve : $(x^4 - 2xy^2 + y^4)dx - (2x^2 y - 4xy^3 + \sin y)\, dy = 0$.

Q.3 (C) Answer any ONE of the following questions. **[03]**

(1) Solve : $y = (1 + p)\, x + p^2$.

(2) Solve : $\sin y \frac{dy}{dx} + x \cos y = x$.

Q.3 (D) Answer any ONE of the following questions. **[05]**

(1) State and prove necessary and sufficient condition for the differential equation $M(x, y)\, dx + N(x, y)\, dy = 0$ to be exact.

(2) Write general form of differential equation of first order and higher degree solvable for p, explain method to solve it.

Q.4 (A) Answer the following questions. **[04]**

(1) Write general form of linear differential equation of order n.

(2) Find C.F. of the differential equation $(D^2 - 1)\, y = \sin x$.

(3) Define : Corresponding equation

(4) Evaluate : $\frac{1}{D} x^2$.

Q.4 (B) Answer any ONE of the following questions. **[02]**

(1) Find P. I. of $\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{4x}$.

(2) Evaluate : $\frac{1}{D^4 - D^2} \sin(2x)$.

Q.4 (C) Answer any ONE of the following questions. **[03]**

(1) Show that $\frac{1}{D^2 + a^2} \cos ax = \frac{x}{2a} \sin ax$.

(2) Solve : $(D^3 - D^2 - 6D)y = (1 + x^2)$.

Q.4 (D) Answer any ONE of the following questions. **[05]**

(1) Show that $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$, Where V is a function of x.

(2) Solve : $(D^2 - 2D + 1)\, y = x^2 e^{3x}$.

Q.5 (A) Answer the following questions. **[04]**

(1) Write general form of homogeneous linear differential equation.

(2) Find auxiliary equation of $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = x^3$.

(3) Find C. F. of $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{2}{x-1}$.

(4) Evaluate : $\frac{1}{\theta^2 + \theta - 2} x^2$.

Q.5 (B) Answer any ONE of the following questions. **[02]**

(1) Express the equation $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{\log x}{x^2}$ in $f(\theta)y = Z$ form.

(2) Solve : $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 9y = 0$.

Q.5 (C) Answer any ONE of the following questions. **[03]**

(1) Solve : $x^2 \frac{d^2y}{dx^2} - 7x \frac{dy}{dx} + 12y = x^4$

(2) In standard notations prove that $\frac{1}{f(\theta)} x^m = \frac{1}{f(m)} x^m$.

Q.5 (D) Answer any ONE of the following questions. **[05]**

(1) Solve : $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} - 6x \frac{dy}{dx} + 6y = \left(x^2 - \frac{1}{x^2}\right)^2$.

(2) Explain a method to find peticular solution of homogeneous linear differential equation with variable coefficient.

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